Universidad de Oviedo

Supermembrane Origin of type IIB Gauged Supergravities in 9D

M. Pilar García del Moral

Universidad de Oviedo

CPAN ingenio 2010

Iberian Strings Bilbao 2012.

OUTLINE

- **Motivation**
  - Gauged supergravities
  - Our Methodology
- **The Building Block**: Supermembrane with Central Charges
- **Tools**
  - The New Mechanism for Gauging a Theory
  - The Global description of (MIM2)
- **Results**: associated Gauged/Massive Supergravities
- Conclusions

Motivation

- ❖ Gauged Supergravities are the most promissing sector of supergravities to model out Phenomenology and Cosmology from Superstring models.
- ❖ It is so far not know the UV completion of these theories. They should correspond to some sectors of M-theory.

GOAL: To obtain the M-theory origin of gauged and massive supergravities in nine dimensions.

(Final Part of my Ph.D student Joselen Peña, Univ. Central de Venezuela)

$$\begin{tikzcd}
\text{Compactified M2}(n=0) \arrow[r, "???"] \arrow[d, "\text{Low Energies}"'] & ??? \arrow[d, "\text{Low Energies}"] \\
\text{Maximal Supergrav. (d)} \arrow[r, "Noether"] & \text{Gauged Supergravities (d)}
\end{tikzcd}$$

Motivation

- Gauge supergravities are deformations of maximal supergravities in lower dimensions. Usually achieved by **Scherk-Schwarz compactifications**, where fields

$$\phi(x^\mu, y) = g_y(\phi(x^\mu))$$

have a nontrivial dependence on the compactified variables, they are not periodic but carry a phase characterized by a monodromy.

$$\mathcal{M}(g) = g(1)g(0)^{-1}$$

y □ y + 1.

But in such a way that the lagrangian only depend on the noncompact variables and the truncation is consistent.

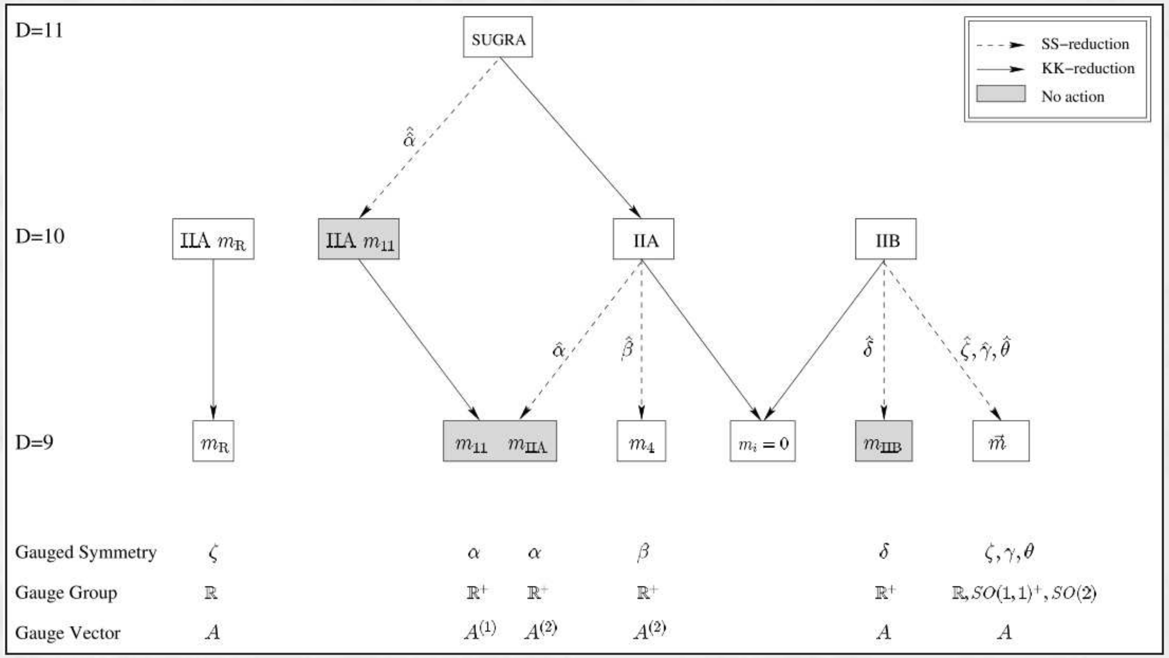
Gauged Supergravities

- Gauged Supergravities have interesting properties as residual global symmetry, nonabelian gauge fields, potentials for scalars..
- Gauge Supergravities and their respective string/M-theory associated to, are characterized by *Monodromies* that carry the deformation with respect maximal supergravities. For Gauged Supergravities in 9D whose global symmetry is SL(2,R)/SL(2,Z) *(Hull 02)* the inequivalent classes of monodromies are

$$\mathcal{M}_h = \begin{pmatrix} e^m & 0 \\ 0 & e^{-m} \end{pmatrix}, \qquad \mathcal{M}_e = \begin{pmatrix} \cos m & \sin m \\ -\sin m & \cos m \end{pmatrix}, \qquad \mathcal{M}_p = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$$

Gauged Supergravities

Diagramme
Taken of the paper
arXiv:hep-th/020920



- **(An Sl(2,Z) multiplet of nine-dimensional type II supergravity theories.**
  Patrick Meessen, *Tomas Ortin*, hep-th/9806120

- Supersymmetry of massive D = 9 supergravity.
  J. Gheerardyn, *P. Meessen*, hep-th/0111130

- **Non-)Abelian Gauged Supergravities in Nine Dimensions** *E. Bergshoeff, T. de Wit, U. Gran, R. Linares, D.Roest* arXiv:hep-th/020920 [Title: Type IIB Seven-brane Solutions from Nine-dimensional Domain Walls E. Bergshoeff, U. Gran, D. Roest arXiv:hep-th/0203202

- **The general gaugings of maximal d=9 supergravity** *J.J. Fernandez-Melgarejo, T. Ortin, E. Torrente Lujan* arXiv:1106.1760

Method

- **Our Starting point** is the compactified supermembrane in nine dimensions as it is the natural sector contained in M-theory :
  - Discover a *New Mechanism for Gauging theories* able to lead to the appropriate monodromies when applied to the case of the compactified supermembrane.
  - Find the *Global description* of the Supermembrane with central charges in terms of nontrivial fiber bundles.

Method

- Check in terms of associated *Monodromies and global dualtiy symmetries* the corresponding gauged supergravities

- *Extra Check*: Agreement with the (p,q) String theories mass spectrum associated to those supergravities, from dimensional reduction of the Supermembrane with central charges.
- Develop the *T-duality transformations in M-theory*, such that they reduce to the ordinary T-duality rules when performed double dimensional reduction.

The Supermembrane

Bergshoeff, Sezgin, Townsend

The Supermembrane

- **Supermembranes** are the natural objects in M-theory. Also called M2-branes (3d objects) and live in 11D Target Space.
  - A **11D M2-brane has continuous spectrum** (de Wit, Luscher, Nicolai) so we do not know how to quantize them. They were interpreted as macroscopic objects. (1997 BFSS Conjecture)
  - The **M2 compactified** in a target space by itself remains unstable due to presence of the string-like spikes tha tdo NOT go away just because of the wrapping. (de Wit, Peeters, Plefka)
  - However THIS is NOT the whole story: there is a sector of the compactified supermembrane (at least) that has **discrete supersymmetric spectrum** (Boulton, MPGM, Restuccia)**: The supermembrane with central charges** (Martin, Restuccia, Torrealba). It admits a microscopical description.

The Supermembrane with Central Charges

Programme from 97: Boulton, MPGM, Martin, Ovalle, Peña, Restuccia, Torrealba,

The hamiltonian of the M2 in the LCG has local and global constrains preserving DPA

$$H = T^{-2/3}\int_\Sigma \sqrt{W}\left[\frac{1}{2}\left(\frac{P_M}{\sqrt{W}}\right)^2 + \frac{T^2}{4}\{X^M, X^N\}^2 + \sqrt{W}\bar{\theta}\Gamma_-\Gamma_m\{X^m, \theta\}\right]$$

$$\phi \equiv d(P_M dX^M + \bar{\theta}\Gamma_-\theta) = 0$$

$$\phi_0 \equiv \int_{\mathcal{C}_s} P_M dX^M + \bar{\theta}\Gamma_- d\theta = 0$$

- The Supermembrane with central charges is a compactified supermembrane

$$\oint_{\mathcal{C}_s} dX = 2\pi R(l_s + m_s\tau)$$

$$dX = dX^1 + i dX^2$$

- with a *EXTRA topological condition* such that its worldvolume carries *Martin-Restuccia monopoles* .

$$\int_\Sigma dX_r \wedge dX_s = n\epsilon^{rs} Area_\Sigma$$

The Supermembrane with Central Charges -2

with a *EXTRA topological condition* such that its worldvolume carries *Martin-Restuccia monopoles .*

$$\int_\Sigma dX_r \wedge dX_s = n\epsilon^{rs} Area_\Sigma$$

$$H = T^{-2/3} \int_\Sigma \sqrt{W}\Big[\frac{1}{2}\Big(\frac{P_m}{\sqrt{W}}\Big)^2 + \frac{1}{2}\Big(\frac{P_r}{\sqrt{W}}\Big)^2 + \frac{T^2}{2}\{X^r, X^m\}^2 \\ + \frac{T^2}{4}\{X^r, X^s\}^2 + \frac{T^2}{4}\{X^m, X^n\}^2\Big] + \text{fermionic terms}$$

$$\sqrt{W} = \frac{1}{2}\epsilon_{rs}\partial_a\widehat{X}^r\partial_b\widehat{X}^s\epsilon^{ab},$$

- **Contains in its spectrum the SL(2,Z) (p,q)-strings IIB AND IIA plus the supermembrane excitations.**

SL(2,Z)symmetries of the MIM2

M.P. Garcia del Moral,I. Martín, A.Restuccia, arXiv:0802.0573

- There are **two types of SL(2,Z)** symmetries $\Sigma \to T^2$

A) SL(2,Z) on the Riemann base

$$d\widehat{X}^r \to S^s_r d\widehat{X}^s \qquad S^s_r \in SL(2,Z)_\Sigma$$

-

$$\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \to \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \begin{pmatrix} S^1_1 & S^1_2 \\ S^2_1 & S^2_2 \end{pmatrix}$$

The MIM2 is invariant under the conformal symmetry homotopic to the identity and diffeomorphisms changing the homology basis

All conformal symmetries of the base are symmetries of MIM2

SL(2,Z)symmetries of the MIM2

M.P. Garcia del Moral,I. Martín, A.Restuccia, arXiv:0802.0573

- **B) SL(2,Z) acting on the target** $\Sigma \rightarrow T^2$

Torus $\mathcal{Z} : z \rightarrow z + 2\pi R(l + m\tau)$

- **fields and the parameters transform as**

$$\begin{aligned} \tau &\rightarrow \frac{a\tau + b}{c\tau + d} \\ R &\rightarrow R|c\tau + d| \\ A &\rightarrow Ae^{i\varphi_\tau} \\ \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} &\rightarrow \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \end{aligned}$$

- **with**

$c\tau + d = |c\tau + d|e^{-i\varphi_t}$ $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2, Z)$

A New Mechanism for Gauging a Theory

María Pilar García del Moral,

arXiv:1107.3255

A New Mechanism for Gauging a Theory

María Pilar García del Moral, arXiv:1107.3255

- Usually we use ***Noether Mechanism*** to gauge a field theory by consistently adding new gauge symmetry to the former action that carries the global symmetry that we want to elevate into a local one.

- The new mechanism I propose is more like a ***Sculpting Mechanism*** in the sense that the new gauge symmetry is obtained by extracting it from the former theory by analizing its global properties.

A New Mechanism for Gauging a Theory

María Pilar García del Moral, arXiv:1107.3255

❖ Noether Mechanism

- *Take for exanple a simple action with a global U(1) symmetry:* $\psi \to e^{i\epsilon}\psi$

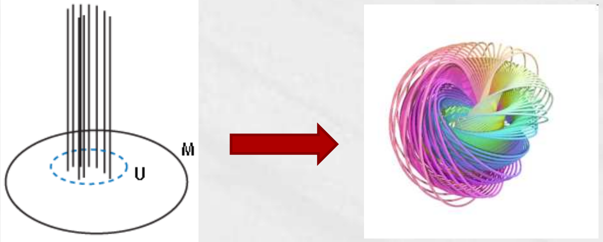
$$S_0 = i\int dx^4 \overline{\psi}\gamma_\mu \partial_\mu \psi$$

- *Impose a local U(1)* $\psi \to e^{i\epsilon(x)}\psi$
- *It requires adding a gauge field* $A_\mu \to A_\mu + \partial_\mu \epsilon(x^\mu)$
- *by replacing the derivative by a covariant derivative and adding terms to achieve the invariance of the action.*

$$S_1 = \int d^4x i\overline{\psi}\gamma^\mu D_\mu \psi$$

Deforming Fibrations

María Pilar García del Moral, arXiv:1107.3255



- The **Sculpting Mechanism** corresponds to **a particular** type of deformation of a nontrivial fibration: The homotopy-type of the base and fiber manifolds are preserved, but it is allowed to change the homotopy-type of the complete fibration.

$$H^2(\Sigma,\mathbb{Z}) \to H^2(\Sigma,\mathbb{Z}_\rho)$$

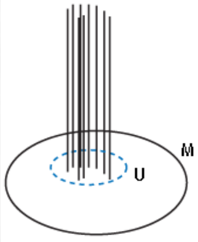
- We will consider as the undeformed fibrations: principal torus fiber bundles with topological invariant Chern class **n** and structure group the **symplectomorphims group.**

$$\int_{\Sigma_2} F_2 = n \in \mathbb{Z} \quad n \neq 0.$$

- One needs closed p-forms and p-cycles to be able to develop this mechanism. In particular for 1-form connection one needs a manifold with one cycles in it.

Deforming Fibrations

María Pilar García del Moral, arXiv:1107.3255



$s_i = 0.$ $t_{ij} = \mathbb{I}$ $\longrightarrow$ $\tilde{t}_{ij} \simeq \xi_{ij}$ $\tilde{s}_i \simeq \Sigma_j;$

A fibration satisfies the cocycle condition and the sections of different patches are expressed in terms of the transition function in the diferents charts.

$$t_{ij}t_{jk}t_{ki} = \mathbb{I} \quad \text{for} \quad U_i \cap U_j \cap U_k \neq \oslash, \quad s_i = \sum_j t_{ij} s_j.$$

- It Changes the following Symmetries:
  - Discrete symmetries $\Gamma \subset G(\mathbb{Z})$
  - Gauge symmetries $\mathcal{A}_r$ with $G_1 = Diff_0^+(B)|_{MCG}$
  - Lagrangian $L = L_0 + \text{Top}$
  - Dynamics : interactions get also modified due to the gauging.

A New Mechanism for Gauging a Theory

Maria Pilar Garcia del Moral, arXiv:1107.3255

Sculpting Mechanism applied to the M2

❖*We need to have a theory with a closed 1-form, and some discrete symmetry G.*

$$\oint_{C_s} dX = 2\pi R(l_s + m_s\tau)$$

❖*We fixed a topological sector that allows us to have a nonvanishing matrix behing harmonic sector.*

$$\int_{\Sigma_2} F_2 = n \in \mathbb{Z} \quad n \neq 0.$$

❖*We perform Hodge decomposition*

$$dX_r = P_r^s d\widehat{X}_s + dA_r$$

A New Mechanism for Gauging a Theory

Maria Pilar Garcia del Moral, arXiv:1107.3255

- *We incorporate the harmonic piece in the definition of a covariant derivative (generalization of weyl)* Global derivative found by *Restuccia,Martin*

$$D_r \bullet = \Theta \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a \widehat{X}_r d_b \bullet$$

- *We define a simplectic covariant derivative*

$$\mathcal{D}_r \bullet = D_r \bullet + \{A_r, \bullet\}$$

- *The gauge transformations are invariant under the symplectomorphism restricted by the discrete symmetries*

$$\delta_\epsilon A_r = \mathcal{D}_r \epsilon$$

A New Mechanism for Gauging a Theory

Maria Pilar Garcia del Moral, arXiv:1107.3255

- *The Gauged supermembrane*

$$\begin{aligned} H = &\int_\Sigma (1/2\sqrt{W})[(P_m)^2 + (\Pi_r)^2 + (1/2)W\{X^m, X^n\}^2 + W(\mathcal{D}_r X^m)^2 + \\ &+ (1/2)W(\mathcal{F}_{rs})^2] + \int_\Sigma [(1/8)\sqrt{W}n^2 - \Lambda(\mathcal{D}_r\Pi_r + \{X^m, P_m\})] + \\ &- (1/4)\int_\Sigma \sqrt{W}n^*\mathcal{F}, \qquad n \neq 0 \end{aligned} \tag{1}$$

$$\int_\Sigma \sqrt{W}[-\bar{\theta}\Gamma_-\Gamma_r\mathcal{D}_r\theta + \bar{\theta}\Gamma_-\Gamma_m\{X^m, \theta\} + \Lambda\{\bar{\theta}\Gamma_-, \theta\}]$$

- *The case usually considered in our papers corresponds to the a Z2xZ2 gauging of the supermembrane and the SO(2) gauged supergravity at low energies.*

$$X_r = \widehat{X}_r + \mathcal{A}_r.$$

$$\mathcal{F}_{rs} = \mathrm{D}_r\mathcal{A}_s - \mathrm{D}_s\mathcal{A}_r + \{\mathcal{A}_r, \mathcal{A}_s\}.$$

A New Mechanism for Gauging a Theory

María Pilar García del Moral, arXiv:1107.3255

- **When to use each one?**
  - If you only need local information, *Noether* is the most appropriate. Supergravity Theories in particular. (Low Energies)

$$\partial_\mu \bullet \to D_\mu \bullet = \partial_\mu \bullet + g\Theta[A_\mu, \bullet]$$

  - Whenever you need global information *Sculpting* is more appropriate. P-brane Theories in general. (High Energies)

$$\partial_r \bullet \to \mathcal{D}_r \bullet = \Theta \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a \widehat{X}_r \partial_b \bullet + \{A_r, \bullet\}$$

Global description of the gauged MIM2

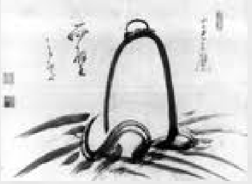
Symplectic torus bundles with monodromy

M.P. Garcia del Moral,I. Martín, J.M. Peña, A.Restuccia. arXiv:1105.3181

- Let us consider a Torus fibration over a torus base manifold. These fibrations with nonvanishing torsion were classified by Khan and Walzack.

$$\xi : F \to E \xrightarrow{p} \Sigma$$

- The topological condition implies a nonvanishing first Chern Class **n**.
- The structure group of the fibration is the group of symplectomorphisms preserving a given class of the symplectic form on the fiber.
- The action of the structure group on the fiber T2 induces an action on the cohomology and homology groups on it.



Symplectic torus bundles with monodromy

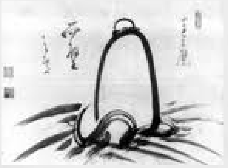
M.P. Garcia del Moral, I. Martín, J.M. Peña, A.Restuccia. arXiv:1105.3181

- The previous hamiltonian is a functional of a symplectic torus bundle over a symplectic base with monodromy $\rho : \pi_1(\Sigma) \rightarrow SL(2, Z)$
- under which the Hamiltonian is invariant

$$\begin{aligned} \tau &\rightarrow \frac{a\tau + b}{c\tau + d} \\ R &\rightarrow R|c\tau + d| \\ A &\rightarrow Ae^{i\varphi_\tau} \\ \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} &\rightarrow \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \end{aligned}$$

- Theorem of Khan: A symplectic form exist in **E** only if the characteristic class **n** is a torsion class in

$$H^2(\Sigma, Z^2_\rho)$$



Symplectic torus bundles with monodromy

M.P. Garcia del Moral,I. Martín, J.M. Peña, A.Restuccia[. arXiv:1105.3181](arXiv:1105.3181)

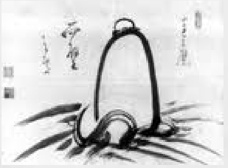
- A example of monodromy with torsion for integers **m,n** differents from zero $\rho : \pi_1(\Sigma) \to SL(2,Z)$

$$\rho(a,b) = \begin{pmatrix} -2mn+1 & 2mn^2+n \\ -m & mn+1 \end{pmatrix}^{(a+b)}$$

- For **m=0** it corresponds to the parabollic monodromy and then it has no torsion .

- The quantization condition of charges, that implies entries of the inequivalent classes of the monodromies to be integers , is not enough characterize fiber bundles with torsion- The classification of these fibration is done according to the Cohomology class

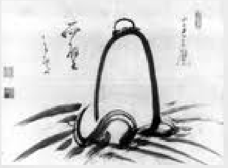
$$H^2(\mathbf{B}, Z^2_\rho) = Z_m \oplus Z_n$$



Symplectic torus bundles with **Extended** monodromy

*M.P. Garcia del Moral, J.M. Peña, A.Restuccia*_arXiv:1202.xxxx

- **Goal**: Find the bundles associated to the M-theory origin of the massive supergravities due to the trombone gauging (supergravities without lagrangian).
- Extensions of the symplectic torus bundle to monodromy groups GL(2,Z) was done by Khan. However GL(2,Z) is defined such that det=+1,-1.
- Dilatations are contained in the group Mat(d,Z) of toral endomorphims whose inverse is defined in the rationals. We follow Cremmer et al. Proposal of active SL(2,Z) symmetries to realize it .
- This extension allows to include monodromies associated to massive supergravities (Trombone supergravities without lagrangian).



Symplectic torus bundles with **Extended** monodromy

M.P. Garcia del Moral, J.M. Peña, A.Restuccia, arXiv:1202.xxxx

- The general compensating transformation for arbitrary τ. in terms of the KK charges and scaling parameter **t**

$$H_{21} = \begin{pmatrix} -\frac{p_j}{q_j}\frac{(p_jq_i-p_iq_j)}{|p_i-q_i\tau|^2}+\frac{q_i}{q_j}t & \frac{p_j}{q_i}+\frac{p_ip_j}{q_iq_j}\frac{(p_jq_i-p_iq_j)}{|p_i-q_i\tau|^2}-\frac{p_i}{q_j}t \\ -\frac{p_iq_i-p_iq_j}{|p_i-q_i\tau|^2} & \frac{q_j}{q_i}+\frac{p_i}{q_i}\frac{(p_jq_i-p_iq_j)}{|p_i-q_i\tau|^2} \end{pmatrix}$$

- The Cohomology of this gauged trombone transformation corresponds to $H^2(\Sigma, Z^2_H)$.
- The transformation on the torus parameters and winding matrix is different ans it corresponds to a inequivalent symplectic torus bundle.

Conclusions

- **Supermembranes with central charges *play* a relevant role inside M-theory/String theory .**
- We have developed a New Mechanism for gauging theories. At High Energies we consider that the natural way to gauge a theory in M-theory corresponds to the ***Sculpting Mechanism*** in order to make contact with Gauged Supergravities at low energies.

$$\begin{tikzcd}
\text{Compactified M2}(n=0) \arrow[r, "\textit{'Sculpting'}"] \arrow[d, "\text{Low Energies}"'] & \text{M2 with central charges } (n \neq 0) \arrow[d, "\text{Low Energies}"] \\
\text{Maximal Supergrav. (d)} \arrow[r, "\textit{Noether}"] & \text{Gauged Supergravities (d)}
\end{tikzcd}$$

Conclusions

- They are symplectic torus bundles over symplectic base spaces associated to principal fiber bundles with non-trivial monodromies that realize the inequivalent classes of gauging of SL(2,Z)

$$H^2(\Sigma, Z^2_\rho)$$

  - At low energies it corresponds to the SL(2,R) parabollic, elliptic and hyperbolic gauged sugras in 9D.

- and the SL(2,Z) active symmetry that model trombone symmetry by inequivalent symplectic torus bundle with nonlinear monodromies

$$H^2(\Sigma, Z^2_H)$$

  - At low energies it corresponds the massive supergravities due to the gauging of the trombone symmetry. .

THANKS!